

## A coupled model for water resources allocation regarding water quality control

S. X. Liu, W. Li, Y. L. Xie, B. Wang and G. H. Huang

### ABSTRACT

In this study, a fuzzy credibility constrained programming (FCCP) model is developed for water quantity and quality management with uncertainties in water quality parameters. The proposed method could reflect not only inexact uncertainties in the objective function, variables and parameters, but also fuzzy uncertainties in the right-hand side. Credibility levels which represent satisfaction degrees of the constraints can be analysed. The developed model is applied to a case study of water resources management within one river basin, three subareas and three water users regarding water environment security. According to the different confidence levels and sewage recovery, scenario analysis is conducted to analyse possible events in water allocation and water quality control. The resulting solutions obtained show that the proposed method can help decision-makers to provide scientific bases for water quantity and quality management and energy system planning, solid waste management and other environmental system problems.

**Key words** | fuzzy credibility constrained programming, optimization model, scenario analysis, uncertainty, water quantity and quality management

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### INTRODUCTION

As one of the most important natural resources, water resources play an important role in human survival, socio-economic sustainable development and ecological environment protection. However, in recent years, owing to the impact of socio-economic rapid development and human activities, water quality deterioration and water shortage have become critical issues in many countries. This situation is particularly serious in many areas mainly located in developing countries. According to United Nations statistics, currently, approximately 700 million people in 43 countries are now suffering from water scarcity, and it is projected that 1.8 billion people will be living in countries or regions with absolute water scarcity by 2025 (UN-Water 2006). Water resources management in China is also facing severe pressure and an effective water resources utilization challenge, and this pressure and challenge mainly originate from water resources and water environmental management strategy. For example, the North China Plain, which is home

to around one-third of China's population, with a significant gross domestic product (GDP) and industrial output, is endowed with less than 8% of the water resources. In contrast, the southwest region has over one-fifth of the country's water resources but produces less than 1% of the national GDP and industrial output (Cheng *et al.* 2009).

Water resources development and utilization faces many serious problems, such as deterioration of the water environment, low water use efficiency and the growing contradiction between supply and demand. In particular, twinned with the rapid increase in water demands driven by municipalities, industry and agriculture, water shortage is exacerbated by the geographically and temporally uneven distribution of precipitation, and surface and groundwater pollution is further deteriorating. In addition, extreme weather occurrences, water infrastructure breakdown and water quality deterioration can also result in the lack of water resources. Regarding the above problems, a

number of studies and measures have been carried out in the last decades to deal with water quantity and quality management problems (Pallottino *et al.* 2005; Wang & Huang 2011; Xie *et al.* 2011; Zhang *et al.* 2011; Wang *et al.* 2012). Among them, many inexact optimization methods, integrated interval, fuzzy and/or stochastic linear programming methods have been developed to deal with uncertainties in water resources management problems (Morgan *et al.* 1993; Huang 1996, 1998; Luo *et al.* 2003; Maqsood *et al.* 2005; Qin *et al.* 2007; Lv *et al.* 2011; Qin & Xu 2011). For instance, Karmakar & Mujumdar (2006) developed a Grey Fuzzy Waste Load Allocation Model, in which uncertainty in the values of membership parameters is quantified in the water quality management model by treating them as interval grey numbers. Zhang *et al.* (2009) proposed a robust chance-constrained fuzzy possibilistic programming model for water quality management within an agricultural system, where solutions for farming areas, manure/fertilizer application amount and livestock husbandry size under different scenarios are obtained and interpreted. Guo *et al.* (2010) developed a fuzzy stochastic two-stage programming approach for supporting water resources management under multiple uncertainties with both fuzzy and random characteristics. Weng *et al.* (2010) developed an integrated scenario-based multi-criteria decision support system for planning water resources management in the Haihe River Basin. Xie *et al.* (2013) developed an inexact two-stage water resources management model for multi-regional water resources planning in the Nansihu Lake Basin, China, and multi-districts, users and water sources were considered in the optimization model. Gema Carmona *et al.* (2013) developed a participatory integrated assessment model, based on the combination of a crop model, an economic model and a participatory Bayesian network, with an application in the middle Guadiana sub-basin, in Spain.

As noted in previous studies, few studies have reported on the presentation and interpretation of multiple uncertainties in parameters during a combined water quantity and quality management system, such as that right-hand side parameters are fuzzy. In practical water quality management problems, many factors have been determined based on decision-makers' subjective judgement rather than the existence of exact numbers, such as parameters of water quality standard. Fuzzy set theory has provided a convenient

formality for classifying water quality conditions, but might easily mislead or bias decision-makers (Silvert 2000; Guler *et al.* 2002; Lu & Lo 2002; Karmakar & Mujumdar 2007; Zhang & Huang 2010). Thus, it is deemed necessary to develop an effective optimization method under fuzzy constraints, which is able to deal with multi-type uncertainties with respect to supporting water quantity management.

Therefore, the objective of this study is to develop a fuzzy credibility constrained programming (FCCP) model based on credibility measures for water quality management and water quantity allocation. The main advantage of the FCCP method is that it can reflect not only inexact uncertainties in the objective function, variables and left-hand side parameters, but also fuzzy uncertainties in the right-hand side. The developed FCCP model is applied to water quality management and water resources allocation problems in a case study that faces severe water quality problems due to industrial pollution and sewage discharge. The results obtained can provide scientific support for regional water quantity and quality management problems under uncertainties at the watershed level.

## FCCP

Credibility constrained programming (CCP), which is based on credibility conception, can be expressed as follows:

$$\text{Maximize } \sum_{j=1}^n c_j x_j \quad (1a)$$

Subject to:

$$\text{Cr} \left\{ \sum_{j=1}^n a_{ij} x_j \leq \tilde{b}_i \quad i = 1, 2, \dots, m \right\} \geq \lambda_i \quad (1b)$$

$$x_j \geq 0, \quad j = 1, \dots, n \quad (1c)$$

where  $x_j = (x_1, x_2, \dots, x_n)$  is a vector of non-fuzzy decision variables;  $c_j$  are cost coefficients;  $a_{ij}$  are technical coefficients;  $\tilde{b}_i$  are right-hand side coefficients;  $\text{Cr}\{\cdot\}$  denotes the credibility of the event  $\{\cdot\}$ ; and  $\lambda$  is the confidence level.

Let  $\xi$  be a fuzzy variable with membership function  $\mu$ , and let  $u$  and  $r$  be real numbers.

The fuzzy variable  $\xi$  is fully determined by the triplet  $(\underline{t}, t, \bar{t})$  of crisp numbers with  $\underline{t} < t < \bar{t}$ , whose membership function is given by

$$\mu(r) = \begin{cases} (r - \underline{t}) / (t - \underline{t}) & \text{if } \underline{t} \leq r \leq t, \\ (\bar{t} - r) / (\bar{t} - t) & \text{if } t \leq r \leq \bar{t}, \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Dubois & Prade (1998) proposed the following indices defined by possibility and necessity measures:

$$\text{Pos}\{\xi \leq r\} = \sup_{u \leq r} \mu(u) \quad (3a)$$

$$\text{Nec}\{\xi \leq r\} = 1 - \text{Pos}\{\xi > r\} = 1 - \sup_{u > r} \mu(u) \quad (3b)$$

where  $\sup(\cdot)$  denotes the largest value of its argument. The credibility measure  $Cr$  is the average of the possibility measure and the necessity measure (Dubois & Prade 1998):

$$Cr\{\xi \leq r\} = \frac{1}{2} (\text{Pos}\{\xi \leq r\} + \text{Nec}\{\xi \leq r\}) \quad (4)$$

From the above definitions, the possibility, necessity and credibility of  $r \leq \xi$  are provided as follows:

$$\text{Pos}\{\xi \leq r\} = \begin{cases} 0 & \text{if } r \leq \underline{t} \\ \frac{r - \underline{t}}{t - \underline{t}} & \text{if } \underline{t} \leq r \leq t \\ 1 & \text{if } r \geq t \end{cases} \quad (5a)$$

$$\text{Nec}\{\xi \leq r\} = \begin{cases} 0 & \text{if } r \leq t \\ \frac{r - t}{\bar{t} - t} & \text{if } t \leq r \leq \bar{t} \\ 1 & \text{if } r \geq \bar{t} \end{cases} \quad (5b)$$

$$Cr\{\xi \leq r\} = \begin{cases} 0 & \text{if } r \leq \underline{t} \\ \frac{r - \underline{t}}{2(t - \underline{t})} & \text{if } \underline{t} \leq r \leq t \\ \frac{2t - \underline{t} - r}{2(t - \underline{t})} & \text{if } t \leq r \leq \bar{t} \\ 1 & \text{if } r \geq \bar{t} \end{cases} \quad (5c)$$

Let  $\sum_{j=1}^n a_{ij}x_j$  be replaced by  $s_i$ . Thus, the constraint (2b) can be represented as:

$$Cr\{s_i \leq \tilde{b}_i, \quad i = 1, \dots, m\} \geq \lambda_i, \quad (6)$$

Normally, a significant credibility level should be greater than 0.5. Therefore, based on the definition of credibility, we have the following equation for each  $1 \geq \mu_{r_i} \geq \lambda_i \geq 0.5$ :

$$\frac{2b_i - \underline{b}_i - s_i}{2(b_i - \underline{b}_i)} \geq \lambda_i \quad (7)$$

where  $\tilde{b}_i$  are right-hand side coefficients fully determined by the triplet  $(\underline{b}_i, b_i, \bar{b}_i)$  of crisp numbers with  $\underline{b}_i < b_i < \bar{b}_i$ , whose membership function is  $\mu$ .

Thus, the CCP can be transformed into an equivalent model as follows:

$$\text{Maximize } \sum_{j=1}^n c_j x_j \quad (8a)$$

Subject to:

$$\sum_{j=1}^n a_{ij} x_j \leq b_i + (1 - 2\lambda_i)(b_i - \underline{b}_i) \quad (8b)$$

$$x_j \geq 0, \quad \forall j \quad (8c)$$

## CASE STUDY

The proposed method is applied to a case study in a water quantity and quality management problem to demonstrate its applicability. To mimic a typical water resource management system, water quality management and water allocation problems are conceptualized in this study, and the case study consists of a system where the major water resource is surface water from a seasonal river. The study system includes three different subareas which are illustrated in Figure 1. Each subarea includes a water intake, a reservoir, a drinking water treatment plant, water consumers and a sewage treatment plant. The water is

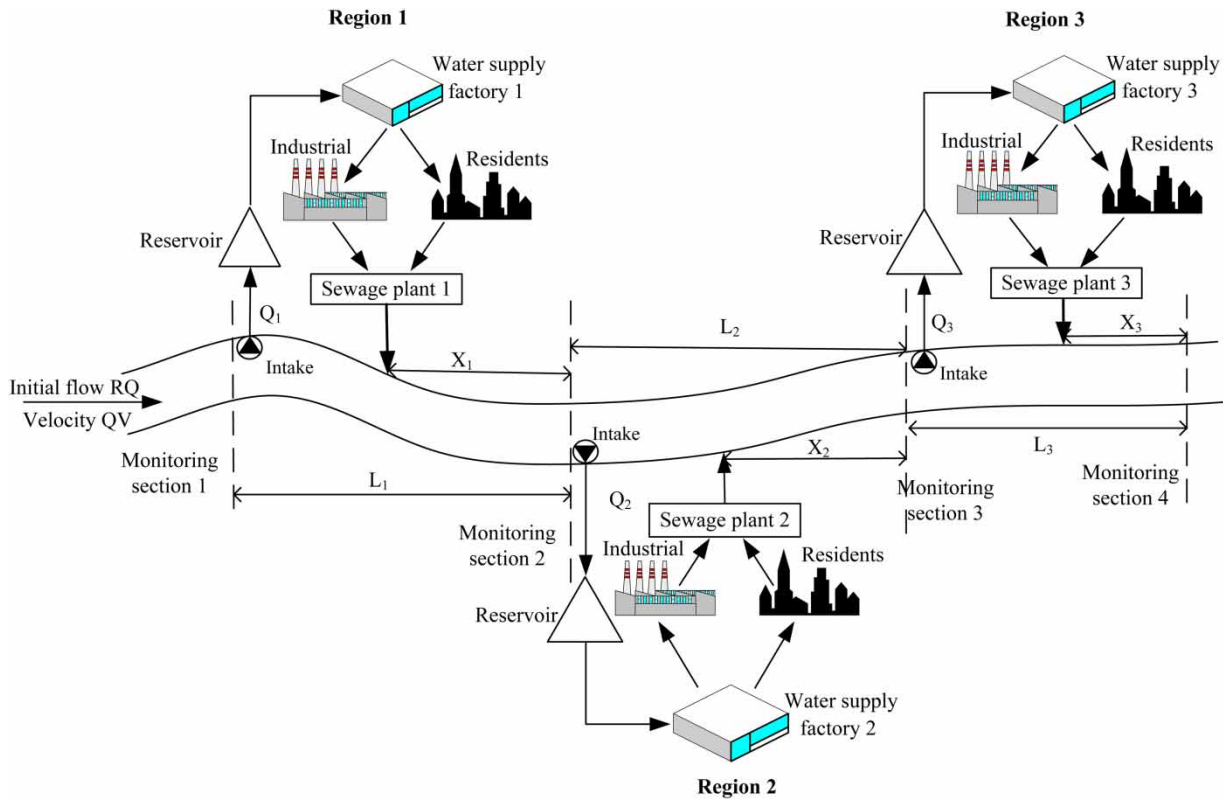


Figure 1 | The study area.

transferred from the intake into the reservoir and then to the treatment plant for disinfection. Finally, the water is distributed to the users which are mainly industrial and residential types. The wastewater generated is transported to the sewage plant. After treatment, part of the water is discharged into the river and part is recycled. Each intake is equipped with such a water quality to ensure that the water quality achieves the standards. Each intake is set in the river flows with a water quality monitoring section in order to ensure the water quality achieves the centralized drinking water standard. In this study, the wastewater treatment technology consists of anaerobic-anoxic-oxic (AAO) process in the first area, an oxidation ditch process in the second area and sequencing in the third area. The sewage water treatment technology is sequencing-batch-reactor (SBR) process in the last area. The parameters of treatment processing efficiency are shown in Table 1.

Water quality in each intake is also affected by various sources upstream. Municipal and industrial

Table 1 | The parameters of treatment processing efficiency

Process	Pollutants	t = 1	t = 2	t = 3	t = 4
AAO process	BOD	0.934	0.943	0.938	0.931
	COD	0.902	0.911	0.91	0.903
	TN	0.73	0.75	0.743	0.726
	TP	0.847	0.86	0.853	0.841
Oxidation ditch process	BOD	0.962	0.973	0.965	0.958
	COD	0.886	0.894	0.89	0.878
	TN	0.654	0.662	0.658	0.65
	TP	0.872	0.886	0.881	0.869
SBR process	BOD	0.898	0.914	0.906	0.896
	COD	0.868	0.874	0.87	0.864
	TN	0.753	0.765	0.76	0.746
	TP	0.912	0.932	0.924	0.904

activities are not only responsible for the water pollution, but also interrelated with each other. Any change in one activity may lead to a series of environmental effects on water resources allocation strategies. Moreover, population growth and economic development in the future

may lead to an increment in water demand and wastewater discharge of each sector. Challenges exist in satisfying the water resources demand and water quality requirement while facilitating regional development. To develop a plan for rational production and economic development, all sectors need to know the amount of water resources that is allocated by the regional manager under environmental requirements. In addition, uncertainties may exist in a variety of impact factors, such as the parameters of water quality standards. Therefore, the problems under consideration are: (1) how to effectively achieve allocation of water to the three regions; and (2) how to maintain the balance between regional development and water environmental protection under a fuzzy water quality standard.

The study time is 1 year, which is further divided into four planning periods according to the seasons. Policies in terms of the related municipal, industrial and agricultural activities, and the wastewater discharges are critical for ensuring maximum system benefit and safe water quality. Generally, the complexities of the study problem include: (1) many parameters are uncertain and are characterized by probabilistic distributions and/or discrete intervals; (2) the objective of maximizing the net benefit can affect the model results and the unbalanced allocation outcome; and (3) dynamic interactions exist between pollutant loading and water quality. The proposed FCCP method is considered suitable for tackling such a problem.

## Model development

Based on the above analysis, it should be an effective measure to plan water resources allocation in the study area with a maximized system benefit under a fuzzy water pollution risk control. The objective of our method is to maximize the net system benefit, which includes (1) benefit of water consumption (BWC), (2) benefit of water reuse consumption (BRW), (3) cost for wastewater treatment operation (CWW), (4) cost for depreciation of sewage treatment equipment (CDS), (5) power expenses for sewage treatment (CPS) and (6) cost for chemical agent consumption of sewage treatment (CCS). The

objective function can be formulated as:

$$\begin{aligned}
 \text{Max BTS} = & \text{BWC} + \text{BRW} - \text{CWW} - \text{CDS} - \text{CPS} - \text{CCS} \\
 & + \sum_{m=1}^3 \sum_{t=1}^4 91 \cdot \frac{\text{INQ}_{mt}}{W_0} + \sum_{m=1}^3 \sum_{t=1}^4 91 \cdot \text{SQ}_{mt} \cdot \text{WP}_t \\
 & + \sum_{m=1}^3 \sum_{t=1}^4 91 \cdot \text{MQ}_{mt} \cdot \text{MP}_t \\
 & + \sum_{m=1}^3 \sum_{t=1}^4 91 \times \xi_{mt} \times Q_{mt} \times \text{RWP}_t \\
 & - \sum_{m=1}^3 \sum_{n=1}^4 \sum_{t=1}^4 91 \cdot k_1 \cdot Q_{mt}^{k_2} \left( \frac{1}{1 - \text{CR}_{mnt}} \right)^{k_3} \\
 & - \sum_{m=1}^M \text{RD}_m \times \left( e^{l+g(F)+\delta(S)} \right) \times (\text{DW}_m)^h \\
 & - \sum_{t=1}^4 \sum_{m=1}^3 91 \cdot \theta \div 10,000 \\
 & \times \left( \frac{3,600 \times \gamma Q_{mt} \text{HP}_m}{1,000 \times \eta_1 \times \eta_2} + \text{EWQ}_{mt} + \text{EEQ}_{mt} \right) \\
 & - \sum_{i=1}^3 \sum_{m=1}^3 \sum_{t=1}^T 91 \times Q_{mt} \times a_{imt} b_{imt} \tag{9a}
 \end{aligned}$$

where the amount of wastewater generated and intake quantity of water can be formulated as follows:

$$Q_{mt} = \text{INQ}_{mt} \cdot \text{IN}_{mt} + \text{MQ}_{mt} \cdot M_{mt} + \text{SQ}_{mt} \cdot S_{mt} \tag{9b}$$

$$\text{WQ}_{mt} = \rho_{mt} \cdot (\text{INQ}_{mt} + \text{MQ}_{mt} + \text{SQ}_{mt}) \tag{9c}$$

where  $m$  is the subareas of the study area,  $m = 1$  for subarea 1,  $m = 2$  for subarea 2 and  $m = 3$  for subarea 3;  $t$  denotes the planning period;  $n$  is the kind of pollutant,  $n = 1$  for biochemical oxygen demand (BOD),  $n = 2$  for chemical oxygen demand (COD),  $n = 3$  for total nitrogen (TN) and  $n = 4$  for total phosphorus (TP);  $\text{INQ}_{mt}$  is the industrial water demand during period  $t$  in subarea  $m$  ( $10^3 \text{ m}^3$ );  $\text{MQ}_{mt}$  denotes the municipal water demand during period  $t$  in subarea  $m$  ( $10^4 \text{ m}^3$ );  $\text{SQ}_{mt}$  is the domestic water demand during period  $t$  in subarea  $m$  ( $10^3 \text{ m}^3$ );  $\text{WQ}_{mt}$  is the intake water quantity during period  $t$  in subarea  $m$  ( $10^3 \text{ m}^3$ );  $W_0$  denotes the water resources consumption of per industrial output value ( $\text{m}^3/\text{RMB}\text{¥} 10^3$ );  $\text{WP}_t$  is the benefit of per domestic water consumption in period  $t$  ( $\text{RMB}\text{¥}/\text{m}^3$ );  $\text{MP}_t$  denotes

the unit benefit of municipal water utilization in period  $t$  (RMB¥/m<sup>3</sup>);  $\xi_{mt}$  is the sewage recycling rate in period  $t$ ; RWP denotes the unit benefit of sewage recycling (RMB ¥/m<sup>3</sup>);  $k_1, k_2, k_3$  are the wastewater treatment operation parameters;  $CI_{mnt}$  is the concentration of pollutant  $n$  in period  $t$  subarea  $m$  (mg/L);  $CR_{mnt}$  denotes the pollutant  $n$  removal efficiency during period  $t$  in subarea  $m$ ;  $Q_{mt}$  is the wastewater amount (10<sup>3</sup> m<sup>3</sup>);  $l, g, \delta, h, F, S$  are sewage treatment equipment depreciation parameters;  $DW_m$  denotes the design capacity of sewage plant in subarea  $m$  (m<sup>3</sup>/day);  $RD_m$  is the fixed assets depreciation rate in subarea  $m$ ;  $EWQ_{mt}$  denotes the power consumption of blower during period  $t$  in subarea  $m$  (KWh);  $EEQ_{mt}$  is the power consumption of other electrical equipment during period  $t$  in subarea  $m$  (KWh);  $\theta$  denotes the electricity price (RMB¥/KWh);  $\gamma$  is the proportion of sewage (N/m<sup>3</sup>);  $HP_m$  denotes the pump total head in subarea  $m$  (m), including primary pump station, secondary pump station and booster pump room;  $\eta_1$  is the pump working efficiency;  $\eta_2$  is the motor working efficiency;  $i$  denotes the kind of agents, including all kinds of chemical reagents, flocculating agent, disinfectant;  $a_{imt}$  is the average usage of potion  $i$  per day during period  $t$  in subarea  $m$  (tonne/10<sup>4</sup> m<sup>3</sup>);  $b_{imt}$  denotes the price of potion  $i$  in period  $t$  (10<sup>3</sup> RMB¥/tonne);  $IN_{mt}$  is the production coefficient of comprehensive industrial wastewater;  $M_{mt}$  denotes the municipal wastewater production factor in period  $t$  subarea  $m$ ;  $S_{mt}$  denotes the sewage generation coefficient during period  $t$  in subarea  $m$ ;  $\rho_{mt}$  is the ratio.

The constraint set consists of some water supply constraints, water demand constraints, capacity constraint, carbon dioxide (CO<sub>2</sub>) emissions constraint and water quality concentration constraints. The information included in the water supply constraints are the potentials and limitations of several water supply options. They permit the withdrawal of water from the river flows though the study area. The water demand constraints describe water resources demand of industrial, municipal and domestic sectors, respectively. Pollutants' removal efficiency constraint considers that the water discharged from the sewage factory must meet the national standards. Sewage treatment plant capacity constraint describes that the wastewater quantity treatment in the sewage plant should be lower than its maximum design capacity. CO<sub>2</sub> emissions' constraint particularly

limits the amount of directly or indirectly carbon dioxide (CO<sub>2</sub>) produced in water treatment from the sewage treatment plant. Water quality concentration constraints describe water quality at the water intake must meet the surface water quality standard of the centralized drinking water resource set by the state. They are delineated sequentially as below.

1. Water supply and demand constraint:

$$\sum_{m=1}^3 WQ_{mt} \leq TAW_t \quad (9d)$$

$$\sum_{m=1}^3 WQ_{mt} \leq TAW_t \quad (9e)$$

$$\sum_{m=1}^3 INQ_{mt} \leq TMIN_t \quad (9f)$$

$$\sum_{m=1}^3 MQ_{mt} \leq TMM_t \quad (9g)$$

$$\sum_{m=1}^3 SQ_{mt} \leq TMS_t \quad (9h)$$

$$INQ_{\min mt} \leq INQ_{mt} \leq INQ_{\max mt} \quad (9i)$$

$$MQ_{\min mt} \leq MQ_{mt} \leq MQ_{\max mt} \quad (9j)$$

$$SQ_{\min mt} \leq SQ_{mt} \leq SQ_{\max mt} \quad (9k)$$

2. Sewage treatment plant capacity constraint:

$$Q_{mt} \leq MAXQ_m \quad (9l)$$

3. Pollutants removal efficiency constraint:

$$CI_{mnt} \cdot (1 - CR_{mnt}) \leq GBSC_n \quad (9m)$$

4. CO<sub>2</sub> emissions constraint:

$$\begin{aligned} & \sum_{m=1}^3 \sum_n^4 91 \cdot \mu \cdot CI_{mnt} \cdot CR_{mnt} \cdot Q_{mt} \\ & + \sum_{m=1}^3 91 \cdot \zeta \cdot \left( \frac{3,600 \times \gamma Q_{mt} HP_m}{1,000 \times \eta_1 \times \eta_2} + EWQ_{mt} + EEQ_{mt} \right) \\ & \leq \text{MAXCQ}_t \end{aligned} \quad (9n)$$

## 5. Water quality concentration constraint:

For monitoring section (2)

$$\begin{aligned} & \text{RCO}_{nt} \cdot \exp\left(\frac{-K_n \cdot L_1}{RV_t}\right) \\ & + \frac{(1 - \xi_{1t}) \cdot Q_{1t} \cdot CI_{1nt}(1 - CR_{1nt})}{RQ_t - WQ_{1t} + (1 - \xi_{1t}) \cdot Q_{1t}} \cdot \exp\left(\frac{-K_n \cdot X_1}{RV_t}\right) \\ & \leq \tilde{S}_{2n} \end{aligned} \quad (9o)$$

For monitoring section (3)

$$\begin{aligned} & \text{RCO}_{nt} \cdot \exp\left(\frac{-K_n \cdot (L_1 + L_2)}{RV_t}\right) \\ & + \frac{(1 - \xi_{1t}) \cdot Q_{1t} \cdot CI_{1nt}(1 - CR_{1nt})}{RQ_t - WQ_{1t} + (1 - \xi_{1t}) \cdot Q_{1t}} \cdot \exp\left(\frac{-K_n \cdot (X_1 + L_2)}{RV_t}\right) \\ & + \frac{(1 - \xi_{2t}) Q_{2t} \cdot CI_{2nt}(1 - CR_{2nt})}{RQ_t - WQ_{1t} + (1 - \xi_{1t}) \cdot Q_{1t} - WQ_{2t} + (1 - \xi_{2t}) \cdot Q_{2t}} \\ & \cdot \exp\left(\frac{-K_n \cdot X_2}{RV_t}\right) \leq \tilde{S}_{3n} \end{aligned} \quad (9p)$$

For monitoring section (4)

$$\begin{aligned} & \text{RCO}_{nt} \cdot \exp\left(\frac{-K_n \cdot (L_1 + L_2 + L_3)}{RV_t}\right) \\ & + \frac{(1 - \xi_{1t}) \cdot Q_{1t} \cdot CI_{1nt}(1 - CR_{1nt})}{RQ_t - WQ_{1t} + (1 - \xi_{1t}) \cdot Q_{1t}} \\ & \cdot \exp\left(\frac{-K_n \cdot (X_1 + L_2 + L_3)}{RV_t}\right) \\ & + \frac{(1 - \xi_{2t}) Q_{2t} \cdot CI_{2nt}(1 - CR_{2nt})}{RQ_t - WQ_{1t} + (1 - \xi_{1t}) \cdot Q_{1t} - WQ_{2t} + (1 - \xi_{2t}) \cdot Q_{2t}} \\ & \cdot \exp\left(\frac{-K_n \cdot (X_2 + L_3)}{RV_t}\right) \\ & + \frac{(1 - \xi_{3t}) \cdot Q_{3t} \cdot CI_{3nt}(1 - CR_{3nt})}{RQ_t - WQ_{1t} + (1 - \xi_{1t}) \cdot Q_{1t} - WQ_{2t} + (1 - \xi_{2t}) \cdot Q_{2t} - WQ_{3t} + (1 - \xi_{3t}) \cdot Q_{3t}} \\ & \cdot \exp\left(\frac{-K_n \cdot X_3}{RV_t}\right) \leq \tilde{S}_{4n} \end{aligned} \quad (9q)$$

where  $TAW_t$  is the available water withdrawals in period  $t$  ( $10^3 \text{ m}^3$ );  $TMIN_t$ ,  $TMM_t$ ,  $TMS_t$  denote the maximum allowable water withdrawals of industry, municipal and domestic sectors in period  $t$  ( $10^3 \text{ m}^3$ );  $INQ_{\min mt}$ ,  $MQ_{\min mt}$ ,  $SQ_{\min mt}$  denote the minimum allowable water withdrawals of industry, municipal and domestic sectors in subarea  $m$  during period  $t$  ( $10^3 \text{ m}^3$ );  $INQ_{\max mt}$ ,  $MQ_{\max mt}$ ,  $SQ_{\max mt}$  denote the maximum allowable water withdrawals of industry, municipal and domestic sectors in subarea  $m$  ( $10^3 \text{ m}^3$ );  $\text{MAXQ}_m$  is maximum design capacity of the wastewater treatment plant in subarea  $m$  ( $10^3 \text{ m}^3$ );  $CR_{mnt}$  is the removal rate of pollutant  $n$  in period  $t$ ;  $GBSC_n$  denotes the pollutant  $n$  discharge standard (mg/L);  $\mu$  is the carbon dioxide ( $\text{CO}_2$ ) emissions estimated coefficients of sewage treatment plant;  $\zeta$  is the  $\text{CO}_2$  conversion coefficient of per units power generation;  $\text{MAXCQ}_t$  denotes the maximum emission quantity of  $\text{CO}_2$  in period  $t$  (tonne);  $RQ_t$  is the river initial flow in period  $t$  ( $\text{m}^3/\text{s}$ );  $RV_t$  is the river flow rate (m/s);  $\text{RCO}_{nt}$  denotes the river initial concentration of pollutant  $n$  in period  $t$  (mg/L);  $\xi_{mt}$  is the recovery rate of sewage in period  $t$ ;  $X_1$  denotes the distance from sewage plant 1 to monitoring section (2) (km);  $L_1$  denotes the distance from monitoring section (1) to monitoring section (2) (km);  $X_2$  is the distance from sewage  $t$  plant 2 to monitoring section (3) (km);  $L_2$  is the distance from monitoring section (2) to monitoring section (3) (km);  $X_3$  is the distance from sewage  $t$  plant 3 to monitoring section (4) (km);  $L_3$  is the distance from monitoring section (3) to monitoring section (4) (km);  $\tilde{S}_{2n}$ ,  $\tilde{S}_{3n}$ , and  $\tilde{S}_{4n}$  are the available concentration of pollutant  $n$  in the centralized drinking water sources in monitoring sections (2)–(4).

## Scenarios' definition

In this study, different water resources allocation schemes for different water users in each region and the concentration control of the pollutants in the monitoring sections can be obtained through changing the confidence level and the recovery rate of wastewater. In this study, the credibility constraints should be at least basically satisfied, and at best, practically satisfied. Then, the confidence level  $\lambda$  is fixed as 0.75, 0.9 and 1, and the recovery rate of sewage  $\xi_{mt}$  is fixed as 30%, 40% and 45%, respectively. Thus, nine

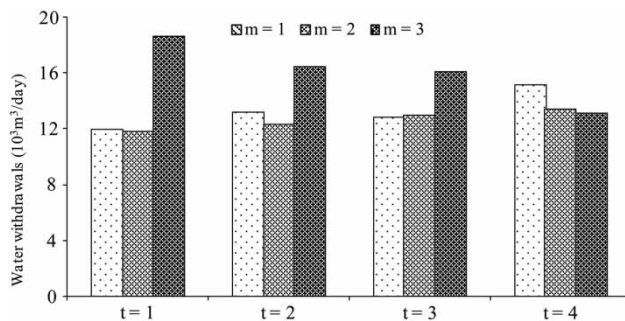
different scenarios could be used to analyse the regional water quantity and quality management problems under different confidence levels and recovery rates of sewage (as shown in Table 2). The planning horizon is divided into four periods throughout 1 year, according to the seasons, and each period takes 1 day to study.

## RESULTS ANALYSIS AND DISCUSSION

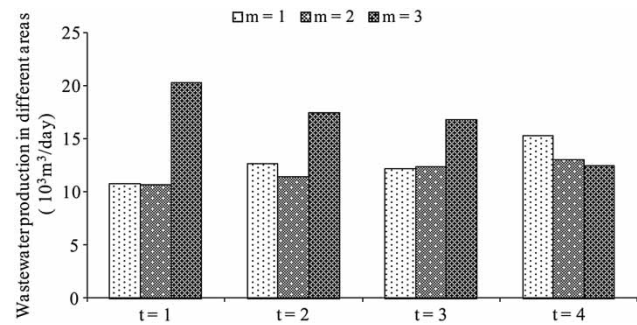
The objective of this model is to maximize the net system benefit, which considers regional water resources allocation as a production problem. The system benefits denote the difference between the economic benefit and the cost for pollution abatement. Figures 2–4 present the optimized wastewater production, water withdrawals and concentration of pollutant BOD and COD of three subareas in scenario 1. Owing to the difference in regional economic development level and water resources demands, and population and regional urban construction scale, water

**Table 2** | Definition of scenarios

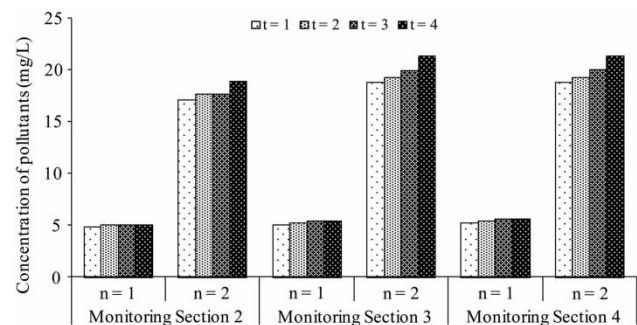
Scenarios	Confidence level	Recovery rate of sewage (%)
1	0.75	30
2	0.75	40
3	0.75	45
4	0.9	30
5	0.9	40
6	0.9	45
7	1.0	30
8	1.0	40
9	1.0	45



**Figure 2** | Optimization of water withdrawals under scenario 1.



**Figure 3** | Optimization of wastewater productions under scenario 1.



**Figure 4** | BOD and COD concentrations in different monitoring sections under scenario 1.

allocation strategies would be different at the regional level and water-user level. Figure 2 illustrates how the water resources from the public river and wastewater recycle allocated to water-users would vary from period 1 to period 4 in different regions. For example, the amount of water resources withdrawals in region 3 follow a decreasing trend from  $18.63$  to  $13.11 \times 10^3 \text{ m}^3/\text{day}$ . This indicated that the water withdrawals would decrease gradually as in region 2 (shown in Figure 1).

Compared with the amount of water withdrawal, the water allocated to region 1 would have a varied trend, and the water withdrawal of region 2 would be stable in general. For example, the water withdrawals in region 1 follow a decreasing trend from  $10.77$  to  $15.37 \times 10^3 \text{ m}^3/\text{day}$  during periods 1–4. The amount of water taken in the third period would be lower than that in the second period.

Figure 3 shows the optimal wastewater emission in the three subareas under scenario 1. Owing to the diverse water withdrawals, the amount of wastewater generated from the industrial and municipal consumers is different in the three regions. For example, in subarea 3, the



wastewater amount decreases from  $20.34$  to  $12.49 \times 10^3 \text{ m}^3/\text{day}$  from periods 1–4. The sewage source mainly includes industrial sewage, life wastewater and municipal wastewater. After treatment, about 30% of sewage would be recycled by industrial, municipal and other sectors, in order to improve the effective utilization of water resources; the other part directly discharges into the river.

Figure 4 presents the BOD and COD concentration in different monitoring sections for the four periods. The initial concentrations of pollutants BOD and COD in monitoring section (1) are lower in order to meet the drinking water standards. After the dilution and degradation process, the concentration of pollutants discharged into the river would be reduced gradually, and the water quality of the river improved accordingly. Pollutant degradation is affected by many factors, such as the original concentration of the pollutants in the rivers, the distance between the plant upstream and downstream of the water inlet, the river flow, etc. For example, at monitoring section (3), the optimized BOD concentrations would be 5.08 mg/L, 5.28 mg/L, 5.46 mg/L and 5.45 mg/L in the four periods, and the concentration of monitoring section (4) would be 5.24 mg/L, 5.42 mg/L, 5.65 mg/L and 5.61 mg/L, respectively. For pollutant COD, the optimized concentrations in monitoring section (1) would be 17.10 mg/L, 17.67 mg/L, 17.67 mg/L and 18.96 mg/L, and at monitoring section (3) it would be 18.86 mg/L, 19.32 mg/L, 20.00 mg/L and 21.40 mg/L,

respectively. It can be found that the optimized BOD and COD concentrations have relatively small growth during the four periods.

Figure 5 shows the amount of water intake and sewage production under different confidence levels, when the recovery rate of sewage is fixed as 40%. Through analysis and comparison it can be found that, at such a rate, the amounts of water intake and wastewater generated tend to stabilize as the confidence level is increased. For example, in subarea 1 during period 2 the water intake is 13.26, 15.02 and  $15.02 \times 10^3 \text{ m}^3/\text{day}$ , in response to the increase in confidence level. On the other hand, the wastewater emission is 15.19, 18.2 and  $18.2 \times 10^3 \text{ m}^3/\text{day}$  corresponding to confidence levels of 0.75, 0.9 and 1, respectively. However, for other periods, the amounts of water withdrawals and wastewater generation in each subarea would show no changes under different confidence levels (e.g., period 1 of subarea 2).

Similar trends can be found in Figure 6, which illustrates the concentration of pollutants BOD and COD at monitoring section (3) under different confidence levels, when the recovery rate is 40%. For example, the BOD concentration remains constant at 4.95 mg/L during period 1 in section (3) for all confidence levels. The changes of COD in each section show a similar behaviour. The analyses indicate that the amount of sewage produced by industrial, municipal and domestic sectors basically remains unchanged under

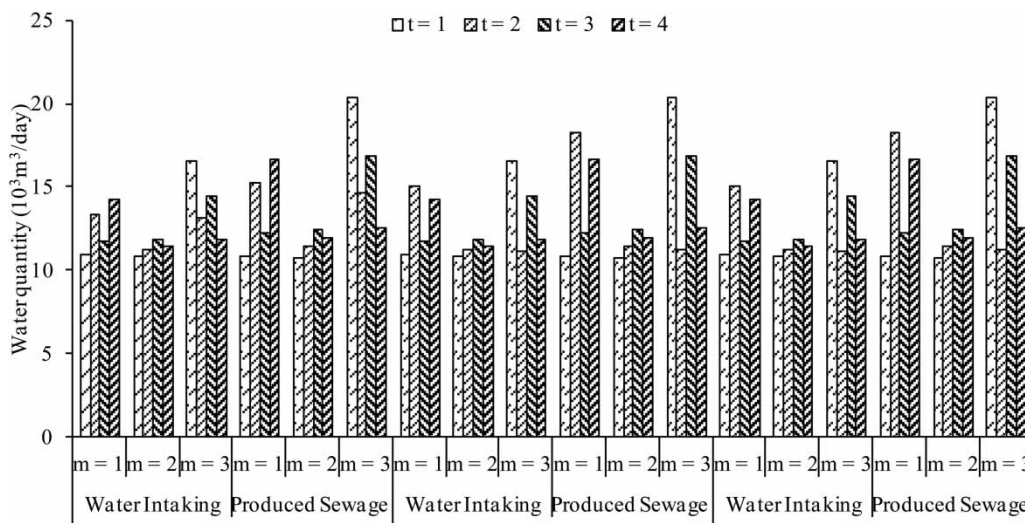


Figure 5 | Amounts of water withdrawals and wastewater emission for a sewage recovery rate of 40% under different confidence levels.

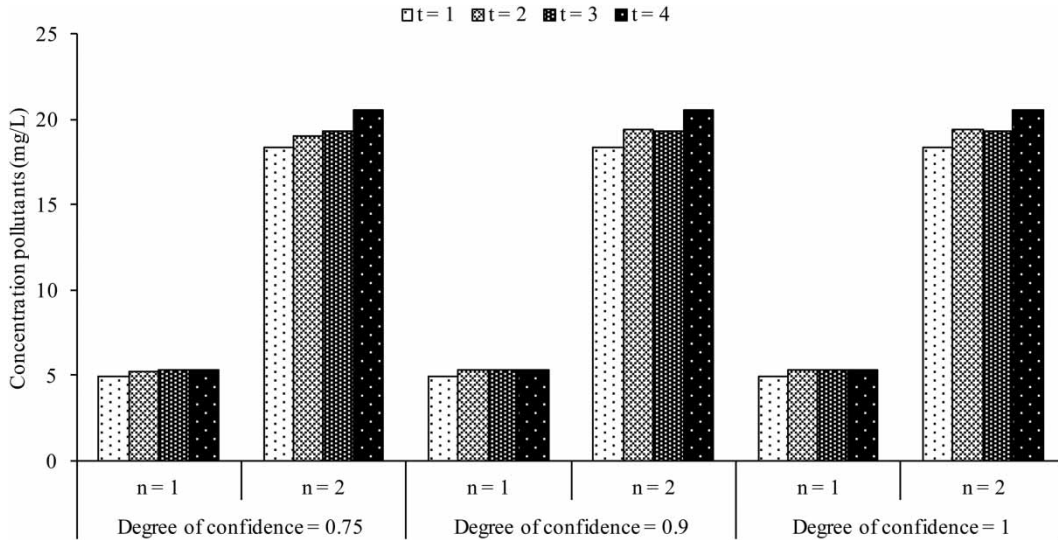


Figure 6 | BOD and COD concentrations in monitoring section (3) for a sewage recovery rate of 40% under different confidence levels.

the condition of maximum system benefit. In addition, when the other factors that affect water quality degradation remain invariant, changing the confidence level has no effect on the concentration of pollutants in the monitoring section.

Figure 7 shows the amount of water intake and sewage discharge under different recovery rates when the confidence level is fixed at 0.75. The figure indicates that in all subareas the intake and sewage quantities are gradually reduced from periods 1 to 4 as the rate of recovery increases. On the other

hand, at period 2 in subarea 1 the quantity of water intake increases gradually while the water quantity of subareas 2 and 3 reduces. For example, in period 2, when the recovery rate is fixed at 30, 40 and 45%, the resulting withdrawals in subarea 1 would be 13.22, 13.26 and 13.27 × 10<sup>3</sup> m<sup>3</sup>/day; for subarea 2, the withdrawals are 12.32, 11.16 and 10.80 × 10<sup>3</sup> m<sup>3</sup>/day; and for subarea 3, the withdrawals are 16.51, 13.06 and 13.09 × 10<sup>3</sup> m<sup>3</sup>/day.

In the case of period 2, a different behaviour is observed. As the recovery rate increases, the sewage quantity in

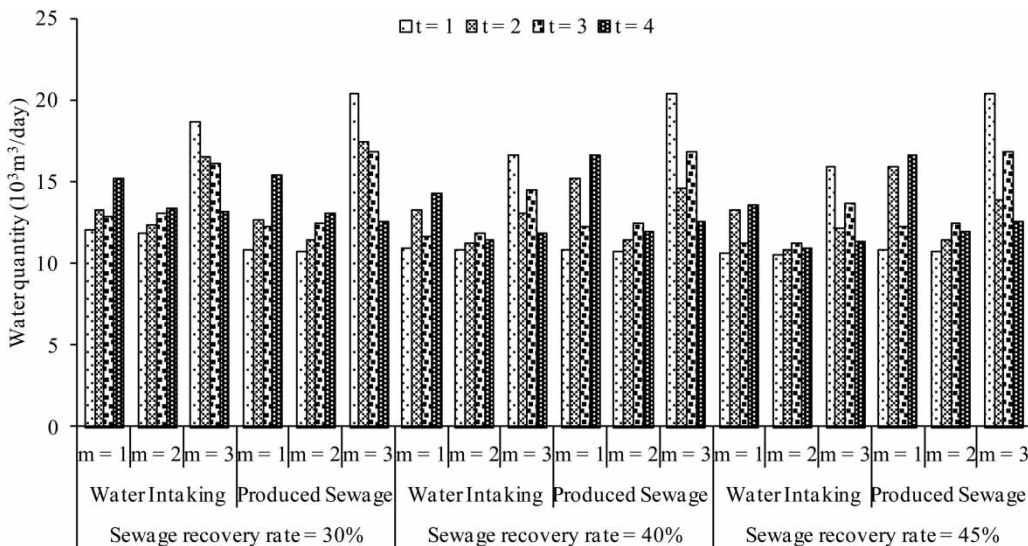


Figure 7 | Amounts of water withdrawals and wastewater emission for a sewage recovery rate of 40% under different recovery rates.

subarea 1 increases gradually. On the other hand, in subarea 3 the quantity is reduced gradually and in subarea 2, the quantity is practically constant across all periods. For example, the sewage quantities of subarea 1 would be  $12.64$ ,  $15.19$  and  $15.86 \times 10^3 \text{ m}^3/\text{day}$ ;  $11.42$ ,  $11.42$  and  $11.42 \times 10^3 \text{ m}^3/\text{day}$  for subarea 2; and  $17.46$ ,  $14.61$  and  $13.85 \times 10^3 \text{ m}^3/\text{day}$  in subarea 3, respectively.

Figure 8 presents the concentration of pollutants BOD and COD in monitoring section (4) under different sewage recovery rates for a confidence level of 0.75. It is observed that as the sewage recovery rate increases, the concentration

of pollutants is reduced gradually in the same period. For example, in period 1, the BOD concentrations are  $5.24 \text{ mg/L}$ ,  $5.11 \text{ mg/L}$  and  $5.07 \text{ mg/L}$  and the COD concentrations are  $18.86 \text{ mg/L}$ ,  $18.35 \text{ mg/L}$  and  $18.20 \text{ mg/L}$  corresponding to sewage recovery rates of 30%, 40% and 45%, respectively.

Figure 9 shows the system net benefits under different recovery rates and confidence levels. Under a fixed confidence level, an increase in the sewage recovery rates produces growth of the net system benefits. For example, when the confidence level is fixed at 0.75, the net benefit

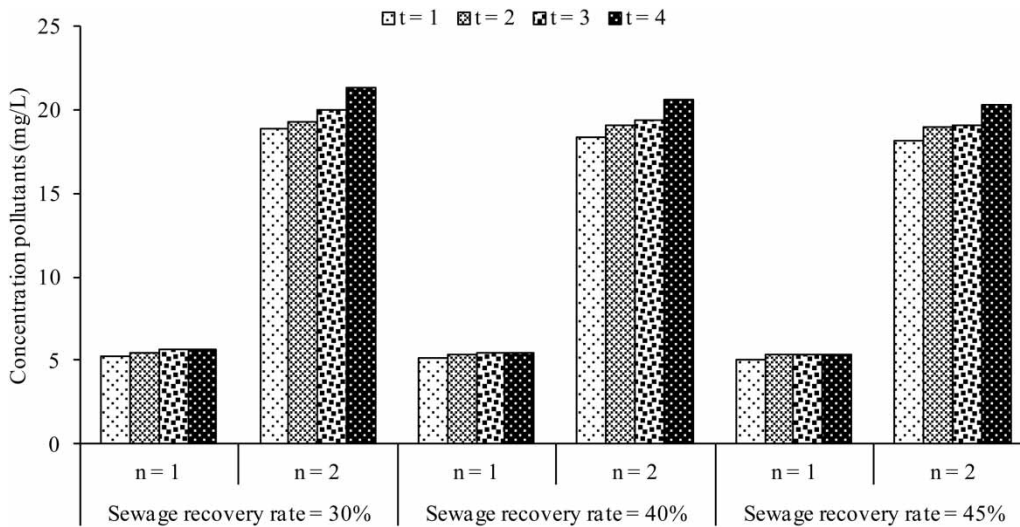


Figure 8 | BOD and COD concentrations in monitoring section (4) under different recovery rates.

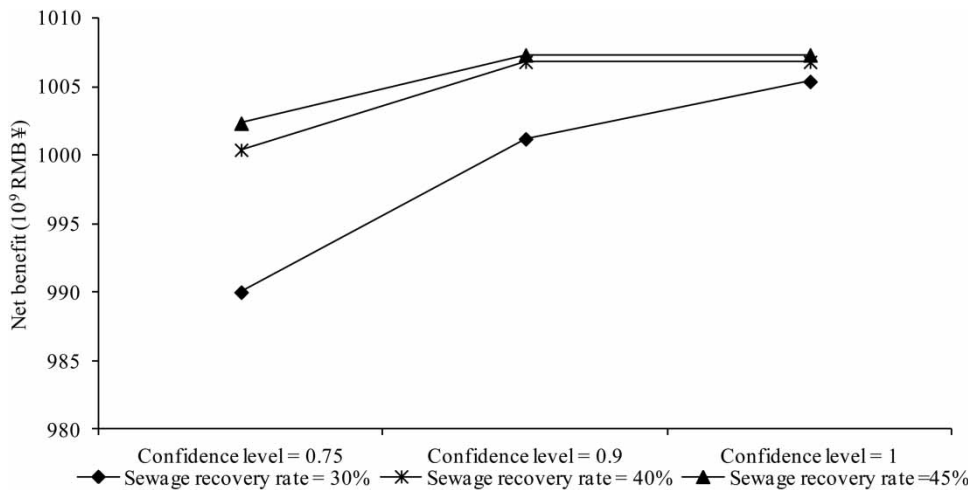


Figure 9 | Net benefits in different scenarios.

of three different recovery scenarios (30%, 40%, 45%) is 990.02, 1000.41 and  $1002.35 \times 10^9$  RMB¥, respectively. The implications are that increasing the recovery rate of the sewage and the confidence level may lead to a higher net benefit and lower sewage treatment costs. Thus, restrictions on the recovery rate of the sewage and the confidence level will significantly affect net system benefits.

## CONCLUSION

In this study, a FCCP method is proposed for water quantity and quality management with respect to planning environmental systems under uncertainty. The developed model was applied to a case study of water resources management consisting of one river basin, three subareas and three groups of users regarding water environment security. A number of scenarios corresponding to different recovery rates and confidence levels were established and examined. The results indicate that our method could help decision-makers generate stable and balanced water resources allocation patterns and water quality management requirements, gain in-depth insights into the effects of uncertainties and analyse trade-offs between system economy and stability. The obtained results indicated that the method is valuable for supporting the adjustment or justification of the existing water quantity and quality management schemes within a complicated water resources system.

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